Test 2A - MTH 1410

Dr. Graham-Squire, Spring 2013

Name: Key

9:00
9:12

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones, computers, and calculators are <u>not</u> allowed on this test.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge.
6. Number of questions = 8. Total Points = 80.

8

1. (~~10~~ points) Find the derivative of $y = x^{3/2}(x^5 - \sqrt{x} + 7x^{-3/2})$. Simplify your answer by combining like terms if necessary.

$y = x^{13/2} - x^{2} + 7$

$$\boxed{y' = \frac{13}{2}x^{11/2} - 2x}$$

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2. (~~10~~ 9 points) Let $g(x)$ be some differentiable function. Use the limit definition of the derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

to prove that $\frac{d}{dx}(5 \cdot g(x)) = 5 \cdot g'(x)$. That is, for $f(x) = 5 \cdot g(x)$, prove that $f'(x) = 5 \cdot g'(x)$.

$$f(x) = 5 \cdot g(x)$$

$$f(x+h) = 5 \cdot g(x+h)$$

$$f'(x) = \lim_{h \to 0} \frac{5 \cdot g(x+h) - 5 \cdot g(x)}{h}$$

$$= 5 \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \boxed{5 \cdot g'(x)}$$

3. (~~10~~ 12 points) Find $\frac{dy}{dx}$ for the equation

$$\frac{d}{dx}\left(\ln(3x^2+y) + e^{(y^2)} = 17x \right)$$

$$\frac{1}{3x^2+y}(6x+y') + e^{(y^2)} \cdot 2y \cdot y' = 17$$

$$\Rightarrow \frac{6x}{3x^2+y} + \frac{y'}{3x^2+y} + 2y e^{(y^2)} y' = 17$$

$$y'\left(\frac{1}{3x^2+y} + 2ye^{(y^2)}\right) = 17 - \frac{6x}{3x^2+y}$$

$$\boxed{y' = \frac{1}{\left(\frac{1}{3x^2+y} + 2y e^{y^2}\right)}\left(17 - \frac{6x}{3x^2+y}\right)}$$

8

4. ($\cancel{10}$ points) Use the quotient rule (or the chain rule) as well as the derivatives for $\sin x$ and $\cos x$ to prove the derivative rule for $y = \csc x$.

$$y = \frac{1}{\sin x}$$

$$y' = \frac{0 \cdot \sin x - \cos x \cdot 1}{\sin^2 x}$$

$$= \frac{-\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$= -\cot x \csc x$$

5. (~~10~~ 12 points) Find the derivative of $f(x) = \left(\frac{\ln x}{x^2}\right)^4$. Simplify your answer, if possible.

$$f'(x) = 4\left(\frac{\ln x}{x^2}\right)^3 \cdot \left(\frac{\frac{1}{x}\cdot x^2 - (\ln x)(2x)}{x^4}\right)$$

$$= 4\left(\frac{\ln x}{x^2}\right)^3 \cdot \left(\frac{\cancel{x}^{1} - 2\cancel{x}\ln x}{x^{\cancel{4}3}}\right)$$

$$= \boxed{4\left(\frac{\ln x}{x^2}\right)^3\left(\frac{1-2\ln x}{x^3}\right)}$$

or

$$\frac{4(\ln x)^3(1-2\ln x)}{x^9}$$

6. ($\cancel{10}$ 8 points) Find the derivative of $y = x \tan\left(\frac{1}{x}\right)$. You do not need to simplify your answer.

$$\boxed{y' = 1 \cdot \tan\left(\frac{1}{x}\right) + x \cdot \sec^2\left(\frac{1}{x}\right) \cdot \left(\frac{-1}{x^2}\right)}$$

7. (~~10~~ 12 points) Find the derivative of

$$f(x) = (\arctan x)\left(\frac{e^x}{\csc x}\right).$$

You do not need to simplify your answer.

$$f'(x) = \frac{1}{1+x^2} \cdot \frac{e^x}{\csc x} + (\arctan x)\left(\frac{e^x \cancel{\csc x} + \cancel{\csc x} \cot x\, e^x}{(\csc x)^2}\right)$$

$$= \frac{e^x}{(1+x^2)(\csc x)} + \arctan x \left(\frac{e^x + (\cot x)\, e^x}{\csc x}\right)$$

8. (~~10~~ 12 points) Find an equation for the tangent line to the curve $y = \sin^3(2x)$ at $x = \frac{\pi}{6}$.

$$x = \frac{\pi}{6} = x_1$$

$$y = (\sin(2x))^3$$

$$f'(x) = y' = 3(\sin(2x))^2 \cdot \cos(2x) \cdot 2$$

$$f'\left(\frac{\pi}{6}\right) = 6\left(\sin\left(\frac{\pi}{3}\right)\right)^2 \cdot \cos\left(\frac{\pi}{3}\right) = 6\left(\frac{\sqrt{3}}{2}\right)^2 \cdot \frac{1}{2}$$

$$= \frac{18}{4} = \frac{9}{2} = m$$

$$f\left(\frac{\pi}{6}\right) = \left(\sin\left(\frac{\pi}{3}\right)\right)^3 = \left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3\sqrt{3}}{8} = y_1$$

$$\Rightarrow \boxed{y - \frac{3\sqrt{3}}{8} = \frac{9}{2}\left(x - \frac{\pi}{6}\right)}$$

Extra Credit(2 points) Describe what derivative rules, and in what order, you would use to find the derivative of

$$\left(\frac{\cos x \ln x}{\tan^4(7x)}\right)^3 \cdot e^x$$

Product Rule, Chain, Quotient Rule. In Process of doing Quotient must do product rule on top and Chain rule, chain rule when taking derivative of bottom.

Test 2B - MTH 1410

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DIRECTIONS

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2. Clearly indicate your answer by putting a box around it.
3. Cell phones, computers, and calculators are <u>not</u> allowed on this test.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge.
6. Number of questions = 8. Total Points = 80.

1. (12 points) Find the derivative of $f(x) = \left(\frac{\ln x}{x^4}\right)^3$. Simplify your answer, if possible.

$$f'(x) = 3\left(\frac{\ln x}{x^4}\right)^2 \cdot \left(\frac{\frac{1}{x}(x^4) - 4x^3 \cdot \ln x}{x^4}\right)$$

$$= 3\left(\frac{\ln x}{x^4}\right)^2 \cdot \left(\frac{x^3 - 4x^3 \ln x}{x^4}\right)$$

$$= \boxed{3\left(\frac{\ln x}{x^4}\right)^2 \cdot \left(\frac{1 - 4\ln x}{x}\right)}$$

or

$$\frac{3(\ln x)^2(1 - 4\ln x}{x^9}$$

2. (12 points) Find an equation for the tangent line to the curve $y = \sin^3(2x)$ at $x = \frac{\pi}{6}$.

$= (\sin(2x))^3$

$y' = 3\,(\sin(2x))^2 \cdot \cos(2x) \cdot 2$

$f'\left(\frac{\pi}{6}\right) = 3\left(\sin\left(\frac{\pi}{3}\right)\right)^2 \cdot \cos\left(\frac{\pi}{3}\right) \cdot 2$

$= 3\left(\frac{3}{4}\right) \cdot \frac{1}{2} \cdot 2$

$= \frac{9}{4}$

$y_1 = f(x_1)$

$= \sin^3\left(\frac{\pi}{3}\right) = \left(\frac{\sqrt{3}}{2}\right)^3$

$= \frac{3\sqrt{3}}{8}$

$\Rightarrow$ tangent line is

$$\boxed{y - \frac{3\sqrt{3}}{8} = \frac{9}{4}\left(x - \frac{\pi}{6}\right)}$$

3. (8 points) Find the derivative of $y = x^{5/2}(x^2 + \sqrt{x} - 4x^{-5/2})$. Simplify your answer by combining like terms if necessary.

$$y = x^{9/2} + x^3 - 4$$

$$\boxed{y' = \frac{9}{2}x^{7/2} + 3x^2}$$

4. (8 points) Let $g(x)$ be some differentiable function. Use the limit definition of the derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

to prove that $\frac{d}{dx}(3 \cdot g(x)) = 3 \cdot g'(x)$. That is, for $f(x) = 3 \cdot g(x)$, prove that $f'(x) = 3 \cdot g'(x)$.

$f(x+h) = 3 \cdot g(x+h)$ $\cancel{f(x+h)=}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3 \cdot g(x+h) - 3 \cdot g(x)}{h}$$

$$= 3 \left(\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} \right)$$

$$= 3 \cdot g'(x)$$

5. (8 points) Use the quotient rule (or the chain rule) as well as the derivatives for $\sin x$ and $\cos x$ to prove the derivative rule for $y = \sec x$.

$$y = \frac{1}{\cos x}$$

$$y' = \frac{0 \cdot \cos x - 1 \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \tan x \cdot \sec x$$

6. (12 points) Find $\frac{dy}{dx}$ for the equation

$$\frac{d}{dx}\left(e^{(y^3)} + \ln(7x^2 + y) = 2x \right)$$

$$\Rightarrow \quad e^{y^3} \cdot 3y^2 \cdot y' + \frac{14x + y'}{7x^2+y} = 2$$

$$\Rightarrow \quad 3y^2 e^{y^3} y' + \frac{14x}{7x^2+y} + \frac{y'}{7x^2+y} = 2$$

$$\left(3y^2 e^{y^3} + \frac{1}{7x^2+y}\right) y' = 2 - \frac{14x}{7x^2+y}$$

$$\boxed{y' = \frac{\left(2 - \frac{14x}{7x^2+y}\right)}{\left(3y^2 e^{y^3} + \frac{1}{7x^2+y}\right)}}$$

7. (12 points) Find the derivative of

$$f(x) = \left(\frac{\sec x}{e^x}\right)(\arctan x).$$

You do not need to simplify your answer.

$$f'(x) = \left(\frac{\sec x \tan x \, e^x - e^x \sec x}{(e^x)^2}\right)(\arctan x) + \left(\frac{\sec x}{e^x}\right) \cdot \frac{1}{1+x^2}$$

or

$$\frac{\sec x}{e^x}\left[(\tan x - 1)(\arctan x) + \frac{1}{1+x^2}\right]$$

8. (8 points) Find the derivative of $y = x \tan\left(\frac{1}{x}\right)$. You do not need to simplify your answer.

$$y' = \boxed{1 \cdot \tan\left(\frac{1}{x}\right) + x \sec^2\left(\frac{1}{x}\right)\left(\frac{-1}{x^2}\right)}$$

or

$$\boxed{\tan\left(\frac{1}{x}\right) - \frac{\sec^2\left(\frac{1}{x}\right)}{x}}$$

Extra Credit (2 points) Describe what derivative rules, and in what order, you would use to find the derivative of

$$\left(\frac{\cos x \ln x}{\tan^4(7x)}\right)^3 \cdot e^x$$

Product Rule first, While doing product must do chain, then quotient rule. While doing quotient rule must do product to top and chain rule, chain rule for derivative of bottom.